Ambiguities in statistical calculations of nuclear fragmentation

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Abstract

The concept of freeze out volume used in many statistical approaches for disassembly of hot nuclei leads to ambiguities. The fragmentation pattern and the momentum distribution (temperature) of the emanated fragments are determined by the phase space at the freeze-out volume where the interaction among the fragments is supposedly frozen out. However, to get coherence with the experimental momentum distribution of the charged particles, one introduces Coulomb acceleration beyond this freeze-out. To be consistent, we investigate the effect of the attractive nuclear force beyond this volume and find that the possible recombination of the fragments alters the physical observables significantly casting doubt on the consistency of the statistical model.

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Multifragmentation of nuclear systems in energetic nuclear collisions serves a novel window for understanding the properties of hot nuclear matter. It has a sensitive bearing on the nuclear equation of state (EoS) [1, 2], focusses our attention on the possibility of liquid-gas phase transition in finite and infinite nuclear systems [3, 4] and from the production of rare isotopes helps for a better understanding of the nucleosynthesis in astrophysical context [5]. Various statistical models have been suggested to explain the phenomenon of nuclear multifragmentation; dynamical models have also been proposed which we will not discuss further in this work[1, 2, 6]. Broadly the statistical approaches are classified in two groups, namely, sequential binary decay (SBD) [7, 8] and one-step prompt multifragmentation (PM) [9, 10]. It is generally believed that at low excitation energy, fragmentation proceeds through SBD whereas at relatively higher energies, it is possibly a one-step break-up process. Nuclear disassembly in the PM picture has been viewed as a statistical process and different genres of statistical hierarchy have been employed to explain the physical process, from grand canonical [11], canonical [10] to micro-canonical [9]. In all these statistical calculations, a freeze-out volume, around $3V_0$ to $8V_0$ (V_0 being the normal nuclear volume of the fragmenting system) is employed when the PM process takes place. The fragments so generated are the primary fragments which are in general in the excited states. Secondary decay from these hot fragments have been taken into account [12, 13]. Furthermore, in the PM models the fragments are generally distributed in the freeze-out volume and Coulomb trajectories are calculated for an improved description of the momentum distribution of the charged fragments. At high excitation energies a collective motion further need to be added to the fragments [1, 2]. These are inconsistent with the assumption of *freeze-out* and is a pointer to the inadequacy of the statistical model. On the same footing, the action of interfragment nuclear force beyond freeze-out needs to be considered. The evolution of the fragments under the combined action of the Coulomb and nuclear fields has not been considered so far except the one reported in ref.[14]. In the exit channel motion, two fragments, when close enough to be under the influence of the nuclear force may recombine to produce an excited heavier fragment which may or may not decay further. In [14], these effects were taken into account and it was found that the yield of relatively heavier fragments is enhanced significantly. Of course this implies that the original yield and momentum distributions given by the statistical model are altered and in order to get, for instance, roughly the original yield, one has to adjust the parameters including the collective flow. The large success of statistical models (without recombination) in reproducing experimental data, has been tacitly assumed as a proof of equilibration in nuclear collisions. Including the nuclear force (which is a must) for such small freeze out volumes might significantly alter this 'idola tribus' (or belief) and the role of dynamics, so far hidden under the carpet of a few parameters and ideal assumptions, must be reconsidered. In this work we will discuss the role of nuclear forces in the model beyond the freeze-out; because we are adding some minimum of dynamical effects, we will dub our approach as Dynamical Statistical Fragmentation Model (DSFM). In a later work we will discuss the flow effects[14, 15]. The situation now is somewhat similar to fission, where one utilizes statistical models to determine the mass distribution at the saddle point and evolves the system including Coulomb, nuclear and even friction forces. At variance with fragmentation, the system evolves after the saddle point thus the mass distributions given by the statistical models are not altered. Including radial flow in DSFM will roughly cancel the effect of recombination and give a picture similar to the fission one.

Isotopic yields from multifragmentation have been employed to infer about important physical observables like the temperature of the fragmenting system [16] and the associated liquid-gas phase transition in finite nuclei. The effect of recombination has so far been ignored in drawing these inferences. The results reported in[14] are restricted to a fixed freeze-out volume V_f and excitation energy E^*; no attempt was made to look into the consequences of the changed isotopic yield on the physical observables after recombination. In the present communication, the effect of recombination with the variation in V_f as well as in E^* has been addressed in some detail and we find that the importance of recombination on the multifragmentation scenario can not be ignored, further adding to the ambiguities of the statistical approaches discussed above.

The model employed in the present calculation is the same as that in[14]. For the sake of completeness, only the salient features of the methodology are discussed here. In the first step, the fragment multiplicities n_i for the various fragments are evaluated in the grand canonical model (GCM). They are given by

$$n_i = V_f \left(\frac{mA_i}{2\pi\hbar^2\beta} \right)^{3/2} \phi_i(\beta) exp[-\beta(B - B_i + V_i - \mu_n N_i - \mu_p Z_i)], \tag{1}$$

where β is the inverse of the temperature T, m the nucleon mass, A_i, N_i and Z_i are the mass, neutron and charge numbers of the fragment species i, B's are the ground state binding energies of the fragmenting system and the generated species, μ's are the nucleonic chemical potentials and $\phi_i(\beta)$'s are the internal partition function. The internal partition function is calculated with the assumption that the excitation of the fragment is below the particle emission threshold. The single particle potential V_i is the sum of the Coulomb and nuclear interaction of the ith fragment with the rest of the fragments and is evaluated in the complementary fragment approximation [17, 18]. Employing the GCM fragment formation probability $p_i = n_i / \sum n_i$, microcanonical events are generated following the method similar to that given by Fai and Randrup [19]. After generation of fragments in a microcanonical event, the fragments are placed in a nonoverlapping manner within the freeze-out volume. The *microcanonical temperature* is evaluated from energy conservation. The fragment velocities are generated from a Maxwell-Boltzmann distribution commensurate with the microcanonical temperature. At this stage the role of the statistical model is over, but a further Coulomb acceleration is now considered which is in contrast to the statistical assumption. Even if the introduction of dynamics in the model is accepted, one should be consistent and include the nuclear forces as well since the nuclear surfaces in the freeze out volume are rather close to each other. Evaluation of the Coulomb interaction is straightforward; the nuclear part of the interfragment interaction in the exit channel is broadly classified in three groups depending on the masses of the fragments. The details are given in ref.[14]. Two fragments in the exit channel are assumed to coalesce when they touch each other. If the excitation energy of the coalesced fragment is above the particle emission threshold (taken as 8 MeV), the fragment is assumed to undergo binary decay; the decay probability is calculated in the transition state model of Swiatecki [20].

To study the effect of recombination in nuclear multifragmentation we have considered ^{197}Au as a representative system. In order to see the effect of recombination on excitation energy, the calculations have been performed at E^*/A=3, 4 and 5 MeV with a fixed freeze-out volume $V_f = 6V_0$. Volume effects have also been considered with E^*/A fixed at 4 MeV. For generation of microcanonical ensemble, typically 10^5 events have been used. Since we have assumed that the fragments are produced in the particle stable states, the charge or mass distribution is decided at the very onset of fragmentation if there is no recombination. The recombined complex may have excitation above the particle emission threshold and they may undergo sequential binary decay in flight till a particle-stable state is reached. In the panels (a), (b) and (c) of Fig.1, the charge distributions at different excitation energies at $V_f = 6V_0$ are displayed. Except for the very light charge particles, the fragment yield is substantially enhanced. At the lowest excitation energy considered (3 MeV/A), the yield of very heavy fragments is found to be somewhat reduced. The neutron yield is enhanced at all the excitation energies considered. This behavior results from a delicate interplay between fragment recombination and subsequent binary decay. It is expected that with reduction in freeze-out volume, the recombination effect would be more prominent. This is apparent from Fig.1(b) and 1(d). This is further evident from the left panel of Fig.2 where the charge distribution has been displayed for a very large freeze-out volume ($16V_0$) at the same excitation energy of 4 MeV per particle. One would expect the recombination effect to be minimal at this large freeze-out volume, however, we find that though it is significantly reduced, it is not negligible, particularly for fragments with $Z > 10$. In order to understand the persistence of the recombination effect at this large freeze-out volume, we have calculated the surface to surface separation (S) of the different fragment pairs (N_{pair}) produced in a disassembly event. In the right panel of Fig.2, the average number of fragment pairs ($\langle N_{pair} \rangle$) present within the separation distance S is displayed for different freeze-out volumes at $E^*/A = 4\ MeV$. The fragment pairs within the nuclear force range (taken as 1.4 fm shown as the horizontal dotted line) are potential candidates to undergo recombination. It is seen from the figure that even at $V_f = 16V_0$, there are significant number of fragment pairs within the nuclear force field.

The knowledge of temperature of the disassembling system is crucial in drawing many important physical inferences such as liquid-gas phase transition. There is no direct way to measure the temperature in such processes; a number of thermometers have been proposed to that end. Experimentally, it has been the usual practice to resort to the isotopic double-ratio [16] to extract the temperature which is based on the statistical multifragmentation model with certain approximations. If the isotopic yield changes due to recombination, the extracted temperature based on this model is bound to be erroneous. To investigate this aspect, we have calculated temperatures from different isotopic double-ratios at a number of excitation energies with and without the effects of recombination. This is displayed in panels (a), (b) and (c) of Fig.3. It is found that the temperatures extracted without recombinaton are consistent with the excitation energies, however, with inclusion of recombination effects, the extracted temperatures from the isotopic double-ratios decrease dramatically. Recombination introduces a multitude of low temperature sources in the system which may be responsible for the reduction in the temperature observed. An anomalous fall in temperature at $E^*/A = 4\ MeV$ is also seen for all the double-ratio thermometers. The temperature extracted after recombination are, however, found to be not too sensitive to the excitation energy(3-6 MeV per nucleon) that we have considered. The dependence of the double-ratio temperature on the freeze-out volume is displayed in Fig.3(d). We have chosen a representative thermometer $(d/t)/(^3He/^4He)$ at an excitation energy $E^*/A = 4\ MeV$. Even at the very large freeze-out volume of $16V_0$, the temperatures extracted without and with recombination effects are appreciably different, but a very slow approach to a common temperature with increasing V_f is apparent from the figure.

To sum up, the effect of recombination of fragments on the charge distributions and isotopic double-ratio temperatures in a nuclear disassembly process in the statistical model has been investigated in this paper at different excitation energies and freeze-out volumes. The effect is found to be very significant for both the observables. With recombination, the yields for relatively heavy fragments are appreciably enhanced at all the excitation energies we have considered. This persistence of larger yield continues even at a freeze-out volume as large as $16V_0$. With recombination considered, the isotopic double-ratio temperature is reduced dramatically. The extracted temperatures without recombination are found to be not too different from those obtained in the Fermi gas model, however, with inclusion of recombination, in the excitation energy range of 3-6 MeV per nucleon that we have investigated, the temperatures are found to be $\sim 4\ MeV$ and not too sensitive to the excitation energy. At relatively higher excitations, the collective flow and an improved dynamics in the fusion process are likely to play a very important role and should be taken into account. We, however, stress that the statistical approaches in a freeze-out scenario should not need any dynamics. The large effects seen with the introduction of dynamics (though has been done in an *ad-hoc* manner) even in a large freeze-out volume is counter-intuitive and cast doubt on the applicability of the freeze-out concept in the fragmentation process; nonequilibrium dynamical features [1, 6] should possibly be incorporated at the very outset.

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Ths DSFM-code discussed here is available and may be requested to one of the authors.

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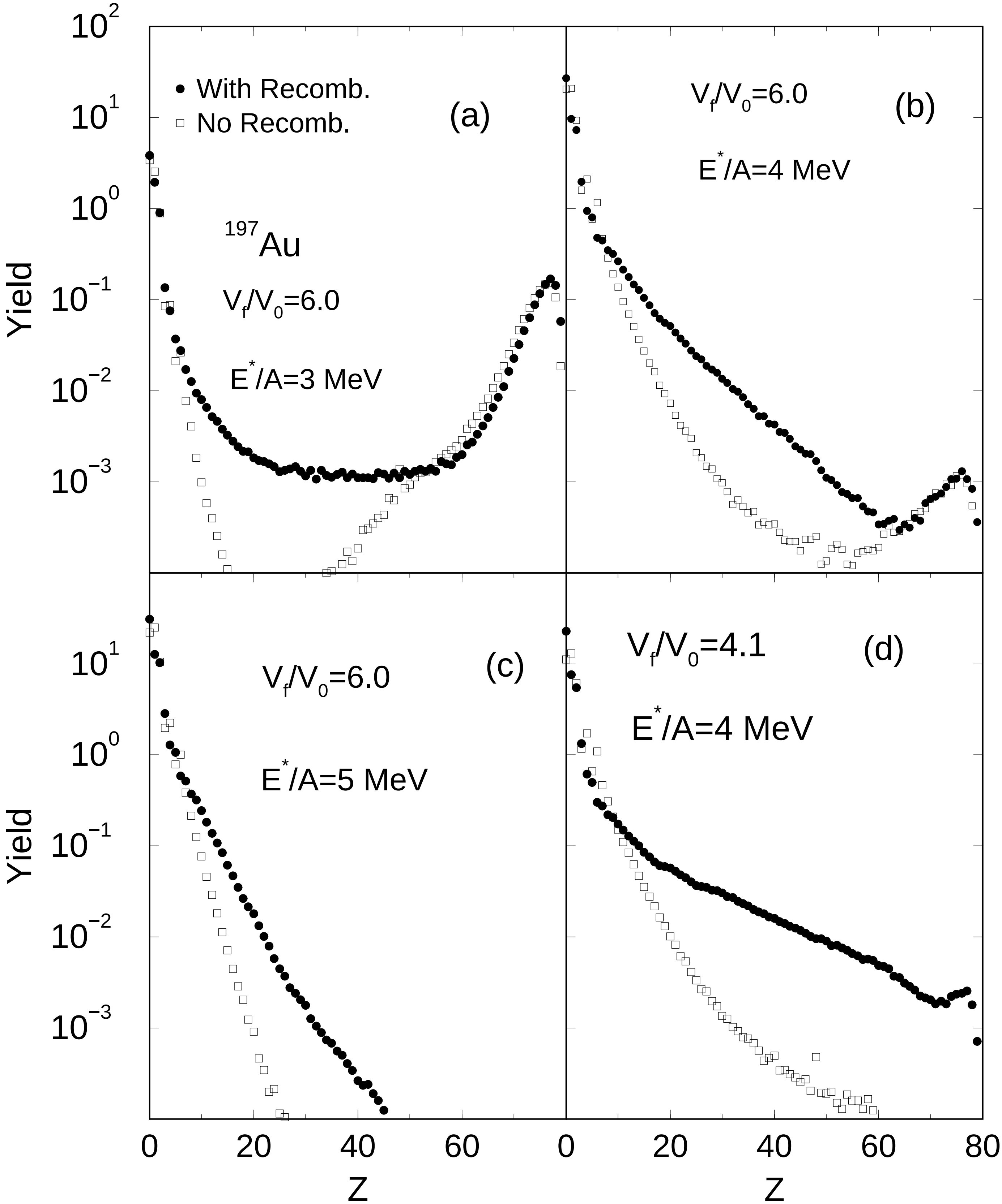
Figure Captions

Fig. 1 Charge distributions from the fragmenting system ^{197}Au with and without recombination at different excitation energies and freeze-out volumes as indicated in the figure.

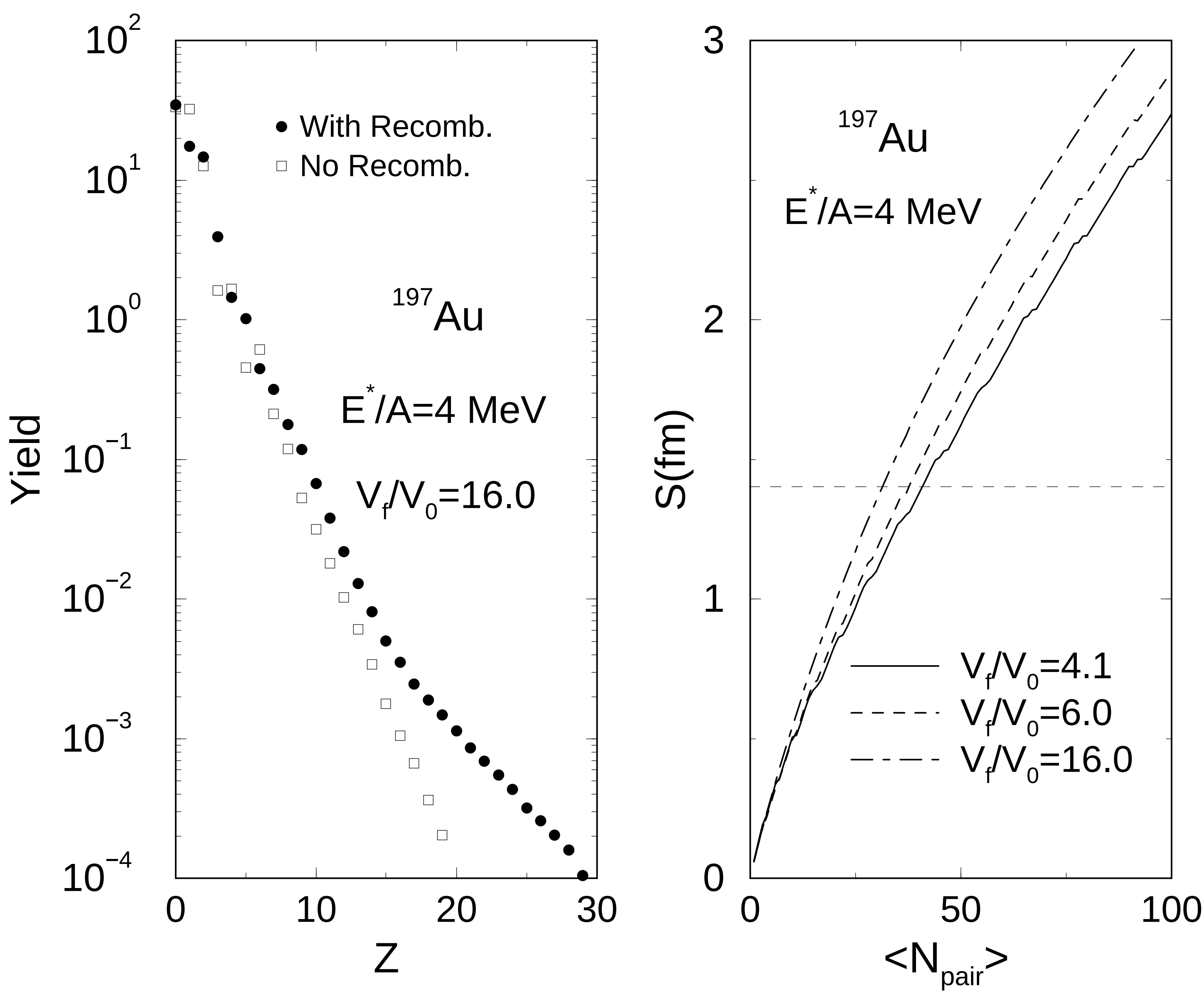
Fig. 2 In the left panel the charge distribution from ^{197}Au at an excitation energy of 4 MeV per nucleon and $V_f = 16V_0$ is displayed. The right panel shows the average number of fragment pairs within a separation distance S at different freeze-out volumes at the same excitation energy.

Fig. 3 In panels (a), (b) and (c), different isotopic double-ratio temperatures are shown at various excitation energies at $V_f = 6V_0$ with and without recombination. In panel (d), the volume dependence of the double-ratio temperature $(d/t)/(^3He/^4He)$ at $E^*/A = 4$ MeV is displayed.



With Recomb.
No Recomb.
(a)
197Au
Vf/V0=6.0
E*/A=3 MeV
(b)
Vf/V0=6.0
E*/A=4 MeV
(c)
Vf/V0=6.0
E*/A=5 MeV
(d)
Vf/V0=4.1
E*/A=4 MeV
Yield
Z

Fig.1



With Recomb.
No Recomb.
197Au
E*/A=4 MeV
Vf/V0=16.0
Yield
Z
S(fm)
<Npair>
Vf/V0=4.1
Vf/V0=6.0
Vf/V0=16.0

Fig.2

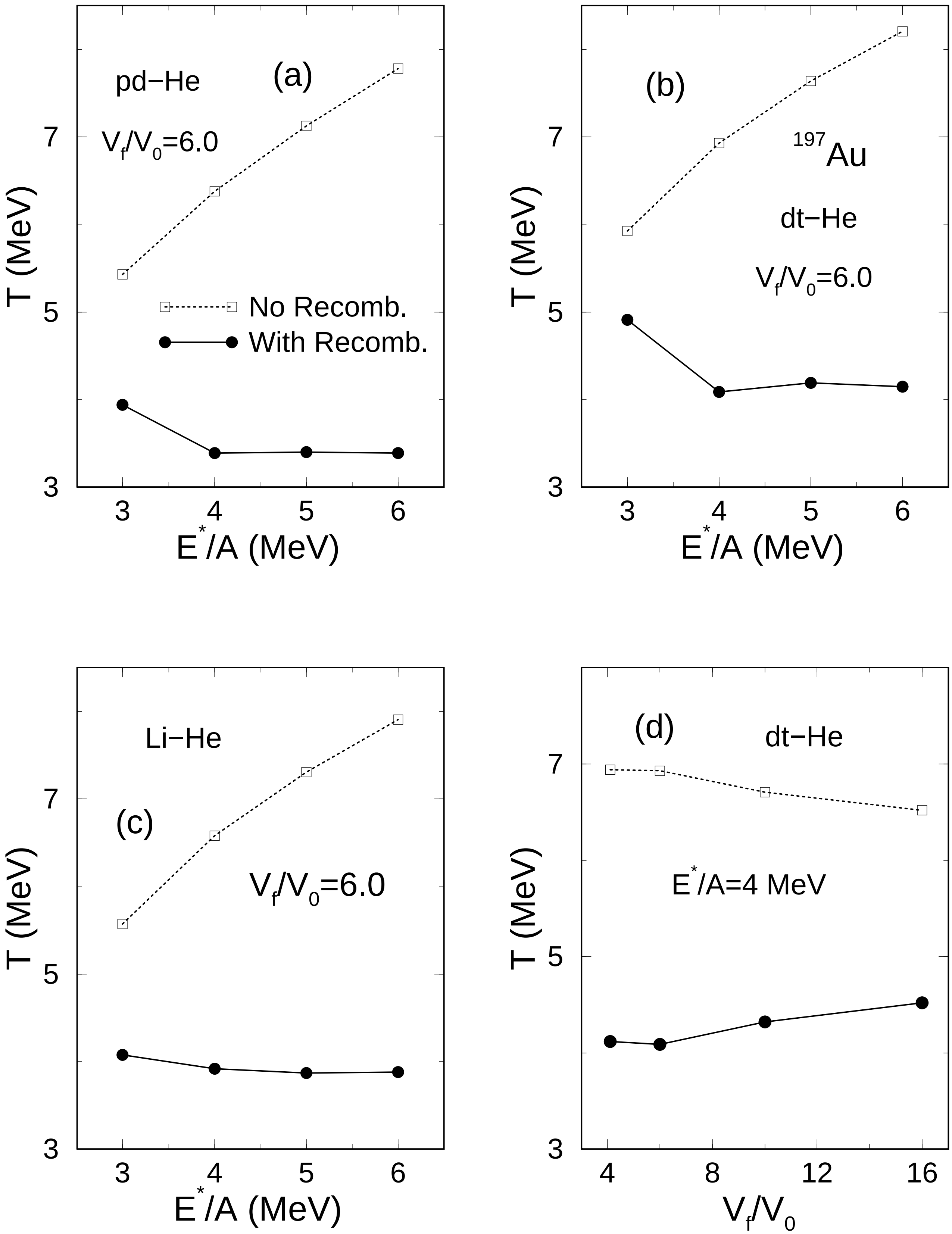

pd−He
(a)
V_f/V_0=6.0
No Recomb.
With Recomb.
T (MeV)
E^*/A (MeV)
(b)
^{197}Au
dt−He
V_f/V_0=6.0
Li−He
(c)
V_f/V_0=6.0
(d)
dt−He
E^*/A=4 MeV
V_f/V_0

Fig.3